KNO scaling in pp / pA and eccentricities in AA collisions

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RIKEN lunch seminar, March 1st, 2012

A.D. + Yasushi Nara,
arXiv:1201.6382

Outline

- (mean) multiplicities from rcBK-UGDs & k_T factorization
- Glauber geometry fluctuations
- “intrinsic” particle production fluctuations: KNO scaling in pp and p+Pb @ LHC
- higher-order eccentricities in A+A
- missing theory:
  - energy dependence of mult. distribution from non-linear evolution ?
  - role of corrections to MV beyond ρ^2 ?

Reminder on (average) dN/dη

rcBK-UGD & k_T factorization works quite decently

rcBK evolution:

basic “degrees of freedom”: dipole scattering amplitude in fund. rep. **(2-point fct)**

$$\mathcal{N}_F(r, Y; b, A) \equiv \frac{1}{N_c} \, \mathrm{tr} \, \langle 1 - V^\dagger(y) V(z) \rangle_Y$$

$$\mathbf{r} = \mathbf{y} - \mathbf{z}$$

BK equation (incl. non-linear terms → saturation of scattering amplitude!)

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \int d^2 r_1 \; K(r, r_1, r_2) \left[\mathcal{N}(r_1, Y) + \mathcal{N}(r_2, Y) - \mathcal{N}(r, Y) - \mathcal{N}(r_1, Y) \mathcal{N}(r_2, Y) \right]$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r_2}$$

running-coupling kernel (Balitsky prescription)

$$K(\mathbf{r}, \mathbf{r_1}, \mathbf{r_2}) = \frac{N_c \, \alpha_s(r^2)}{2\pi^2} \left[\frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2 \, r_2^2} + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

dipole scattering amplitude in adj. rep.

$$\mathcal{N}_A = 2 \, \mathcal{N}_F - \mathcal{N}_F^2$$

$k_\perp$-factorization, multiplicity in A+B --> g+X

unintegrated gluon distribution:

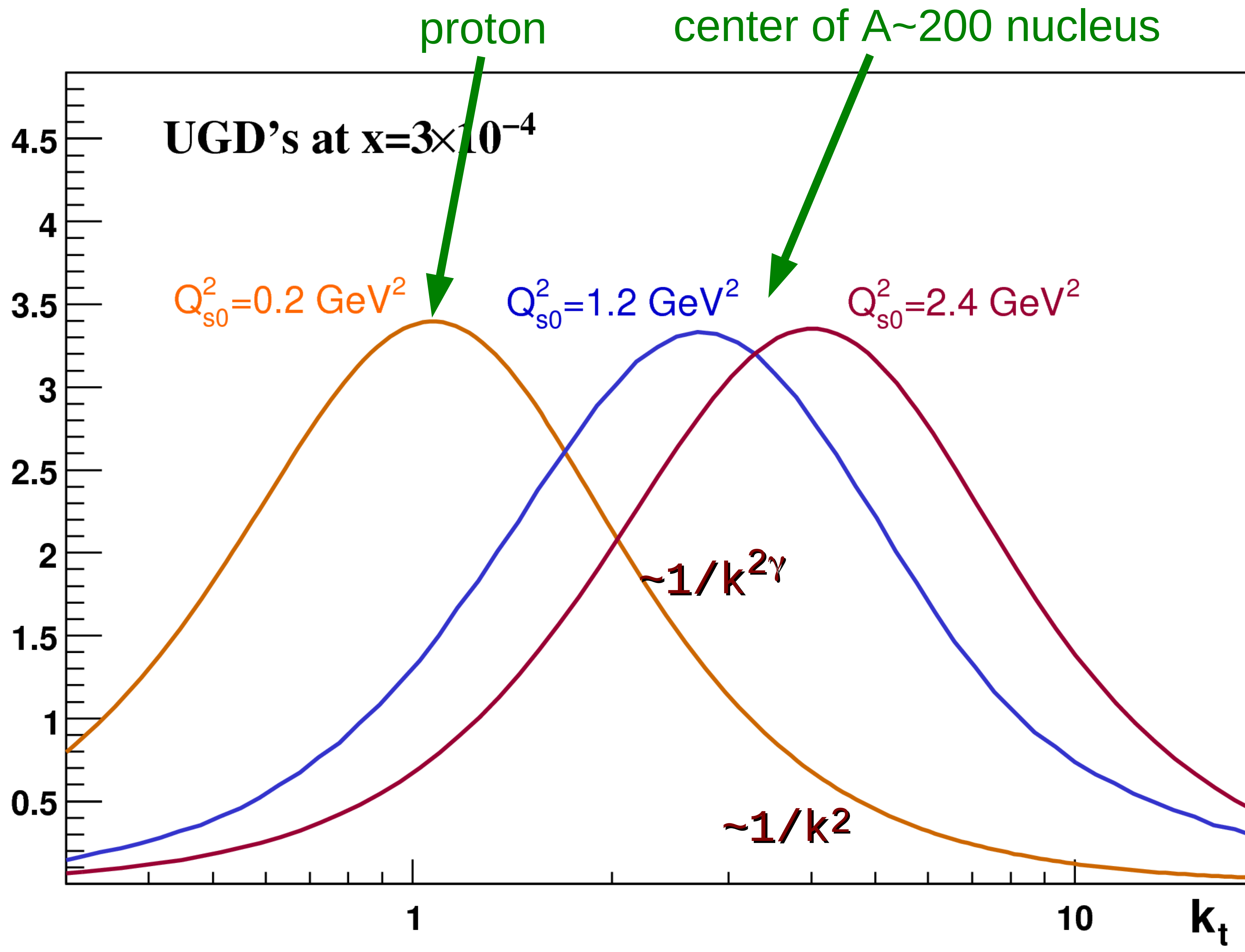
$$\varphi(k,Y;b,A) = \frac{C_F\,k^2}{\alpha_s(k)} \int \frac{d^2\mathbf{r}}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{r}} \mathcal{N}_A(r,Y;b,A)$$

multiplicity (Kharzeev & Levin):

$$\frac{dN^{A+B\to g}}{dy\,d^2b} = K\,\frac{1}{2C_F}\int\frac{d^2p_t}{p_t^2}\int^{p_t} d^2k_t \\ \alpha_s(Q)\,\varphi\left(\frac{|p_t+k_t|}{2},x_1\right)\,\varphi\left(\frac{|p_t-k_t|}{2},x_2\right)$$

- finite at $p_t \to 0$ if UGD does not blow up
- $x_{1,2} = (p_t/\sqrt{s})\exp(\pm y)$; $Y_{1,2}=\log(x_0/x_{1,2})$
  where $x_0=0.01$ is assumed onset of rcBK evol.
- K = 1.5 – 2, appears reasonable

uGD at $x = 3x10^{-4}$ (e.g. pt=2GeV, y=0, √s=7TeV)



what is the initial condition for rcBK evolution ?

- don't really know, small-x doesn't tell
- needs to be set at “sufficiently” small x_0 so that rcBK can take it from there; in practice, x_0=0.01 ?
- for large $A^{1/3}$, MV model should provide initial condition:

$$\mathcal{N}_F(r, Y=0; b) = 1 - \exp\left[-\frac{r^2\, Q_{s0}^2(b)}{4}\,\ln\left(\frac{1}{\Lambda\, r} + e\right)\right]$$

McLerran-Venugopalan action (for large nucleus, $A^{1/3} \to \infty$)

$$S_{\rm MV} = \int d^2x_\perp \frac{1}{2\mu^2}\rho^a\rho^a$$

+ soft YM fields + coupling of soft ↔ hard

- $\mu^2 \sim g^2 A^{1/3}$

$$\left\langle 1 - \frac{1}{N_c}\,\mathrm{tr}\, V_x V_y^\dagger \right\rangle_{\rm MV} = 1 - \exp\left[-\frac{1}{4} r^2 Q_{s0}^2 \log\frac{1}{r\Lambda}\right]$$

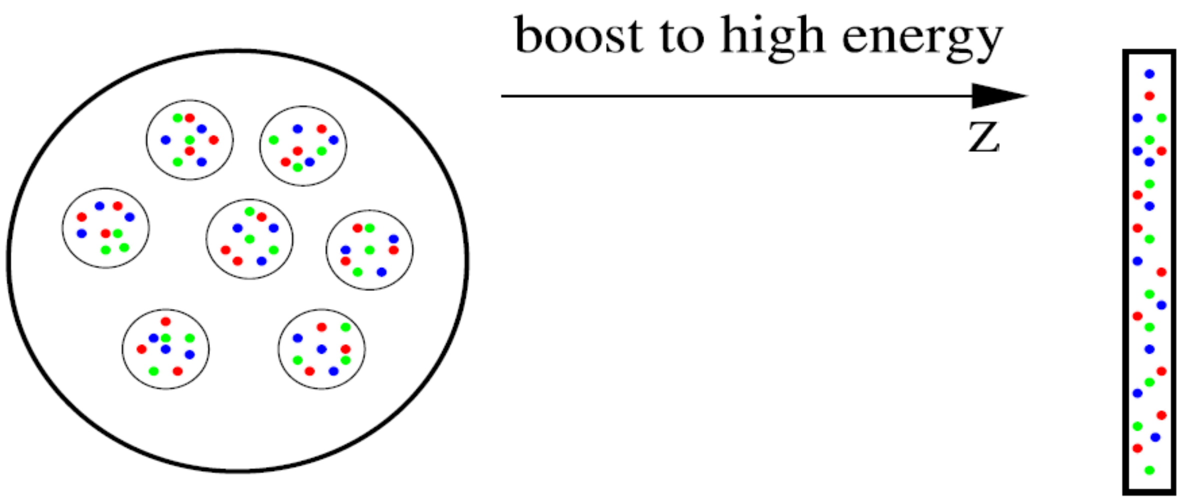
(in log (1/rΛ) » 1 limit)

- in what follows

  MV: $Q^2_{s0,N} = 0.2\ \mathrm{GeV}^2$

- for nucleus, at transv. position b:

  $Q^2_{s0}(b)$ = (# nucleons at b) X $Q^2_{s0,N}$

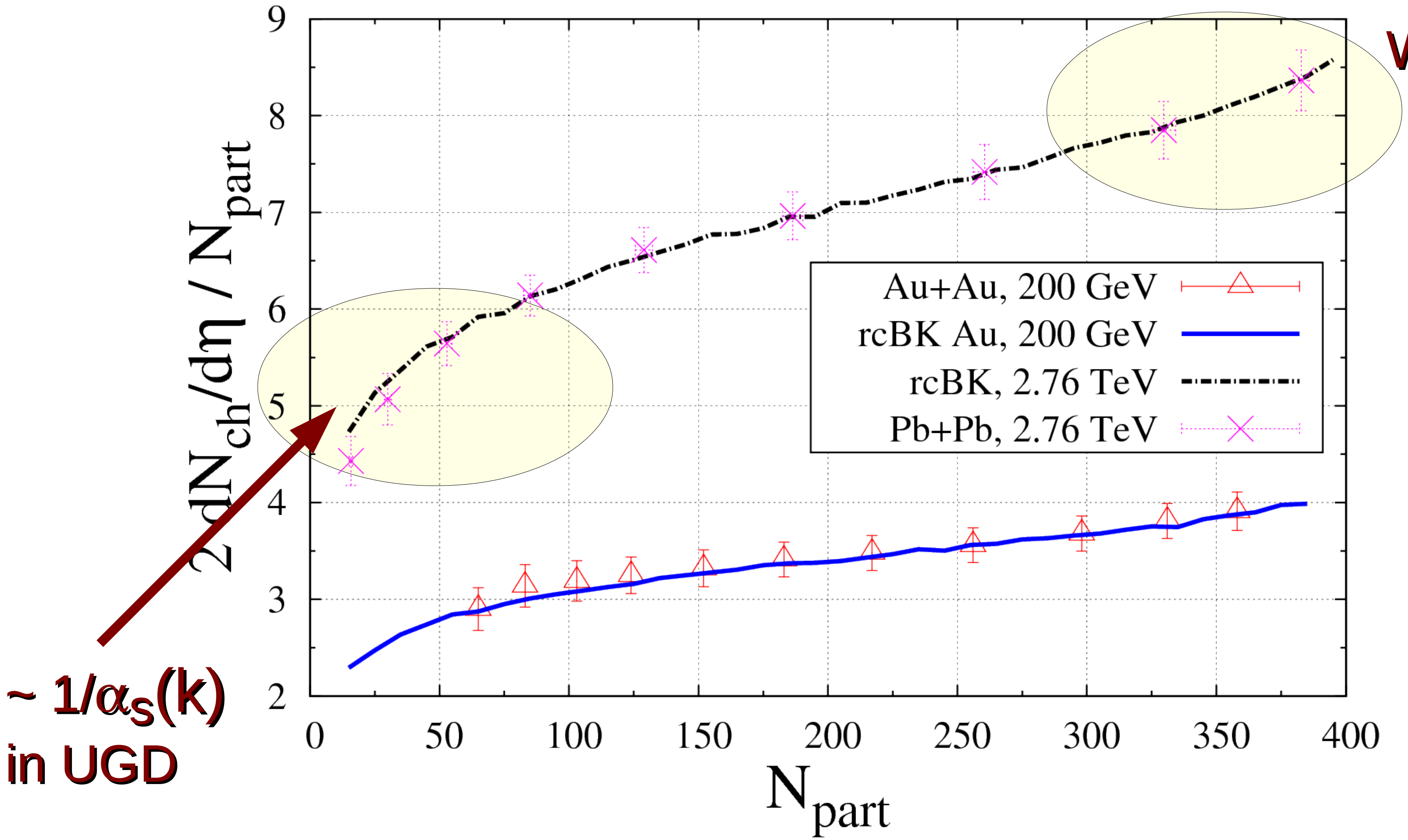


McLerran & Venugopalan

side view

AA : centrality and energy dependence of multiplicities

Albacete & Dumitru: arXiv:1011.5161



- assumes $N_{hadr} \sim N_{glue}$

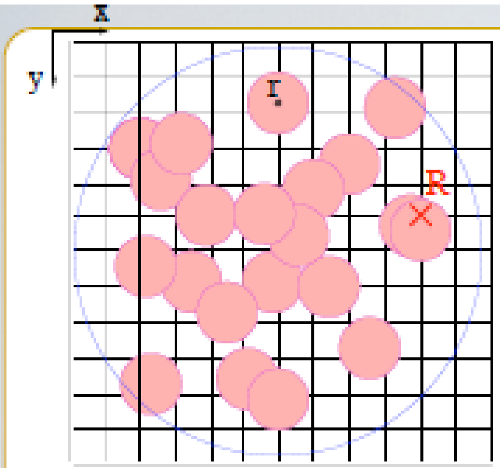
Glauber / geometry fluctuations

fluctuations of transverse positions of valence charges in the colliding nuclei

(can be treated in CGC framework due to separation of slow & fast variables introduced by MV !)

first “MC-KLN” model by H. Drescher & Y. Nara, 2007

fluctuations of valence partons in ⊥ plane



1. Initial conditions for the evolution (x=0.01)

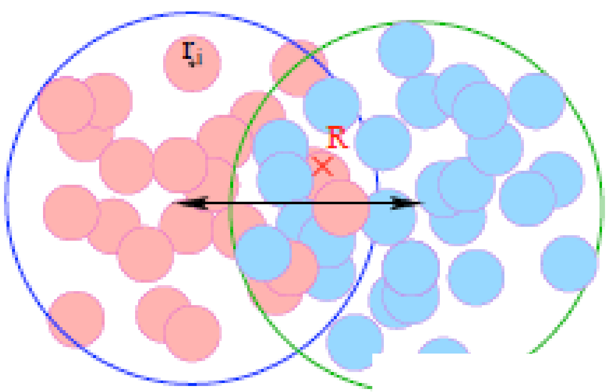
$$N(\mathbf{R}) = \sum_{i=1}^{A} \Theta\left(\sqrt{\frac{\sigma_0}{\pi}} - |\mathbf{R} - \mathbf{r_i}|\right) \longrightarrow Q^2_{s0}(\mathbf{R}) = N(\mathbf{R})\, Q^2_{s0,\,\mathrm{nucl}}$$

$$\varphi(x_0 = 0.01, k_t, R)$$

2. Solve local running coupling BK evolution at each transverse point

rcBK equation or KLN model

$$\varphi(x, k_t, R)$$



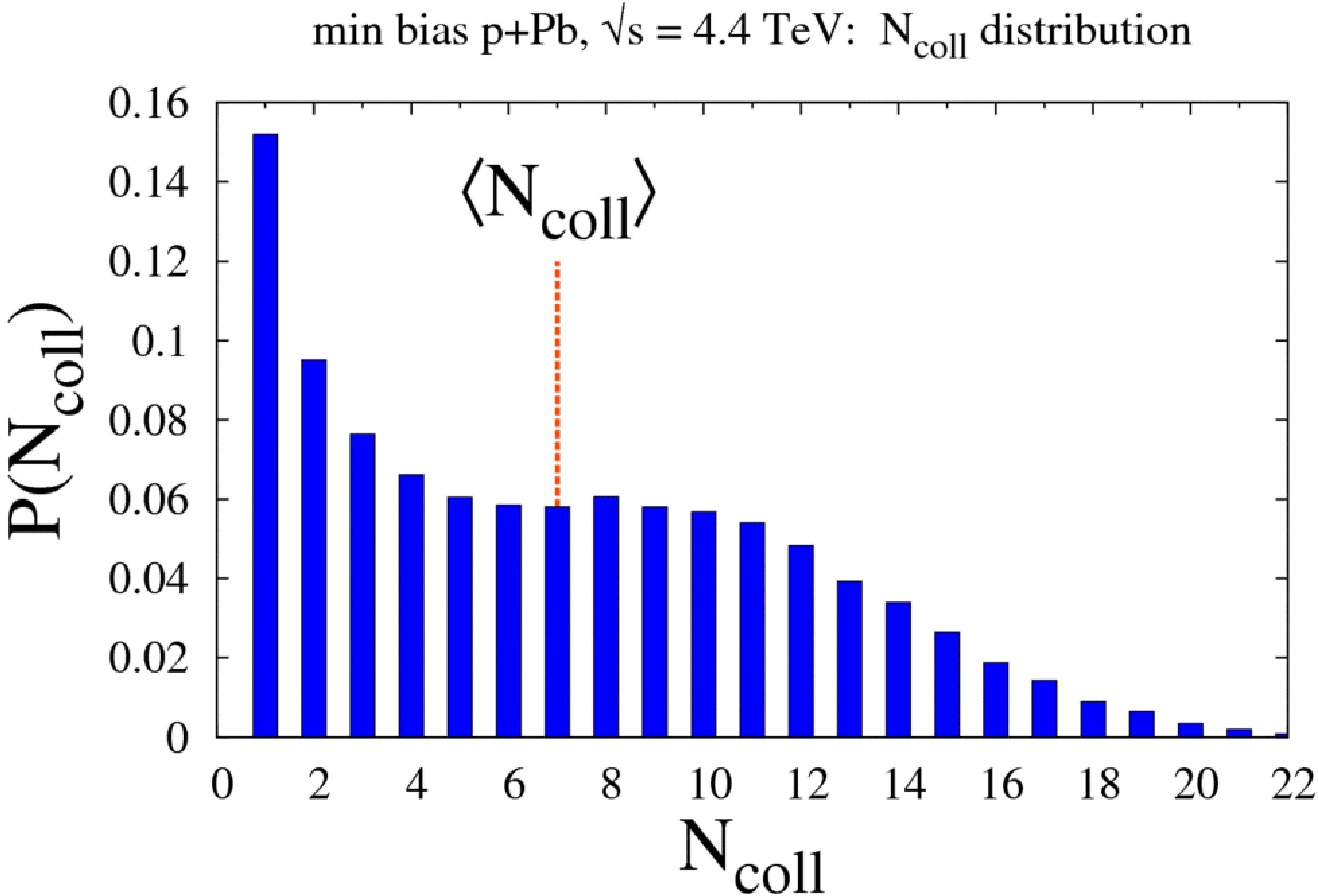
3 Calculate gluon production at each transverse point according to kt-factorization

INPUT: $\varphi(\mathbf{x} = \mathbf{0.01}, \mathbf{k_t})$ FOR A SINGLE NUCLEON:

$$N_{\mathrm{part,A}}(\vec{b}) = \sum_{i=1\cdots A} \Theta\left(P(\vec{b} - \vec{r}_i) - \nu_i\right) .$$

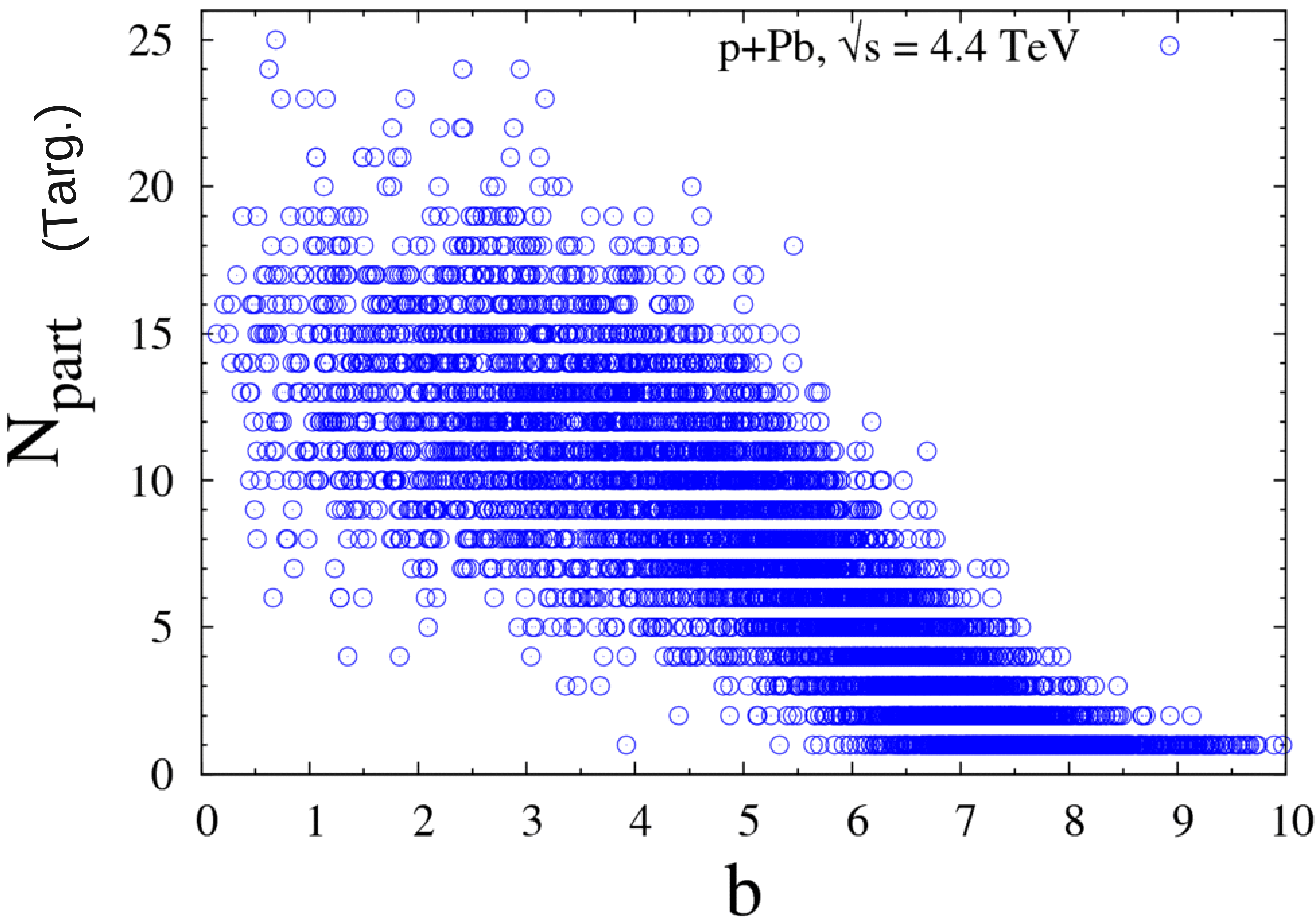
$$P(b) = 1 - \exp[-\sigma_g T_{pp}(b)], \qquad T_{pp}(b) = \int d^2 s\, T_p(s)\, T_p(s-b)$$

$$T_p(r) = \frac{1}{2\pi B} \exp[-r^2/(2B)] \qquad \sigma_{NN}(\sqrt{s}) = \int d^2 b\, (1 - \exp[-\sigma_g T_{pp}(b)])$$



min bias p+Pb, √s = 4.4 TeV: N_{coll} distribution
$\langle N_{coll} \rangle$
$P(N_{coll})$
N_{coll}
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 2 4 6 8 10 12 14 16 18 20 22

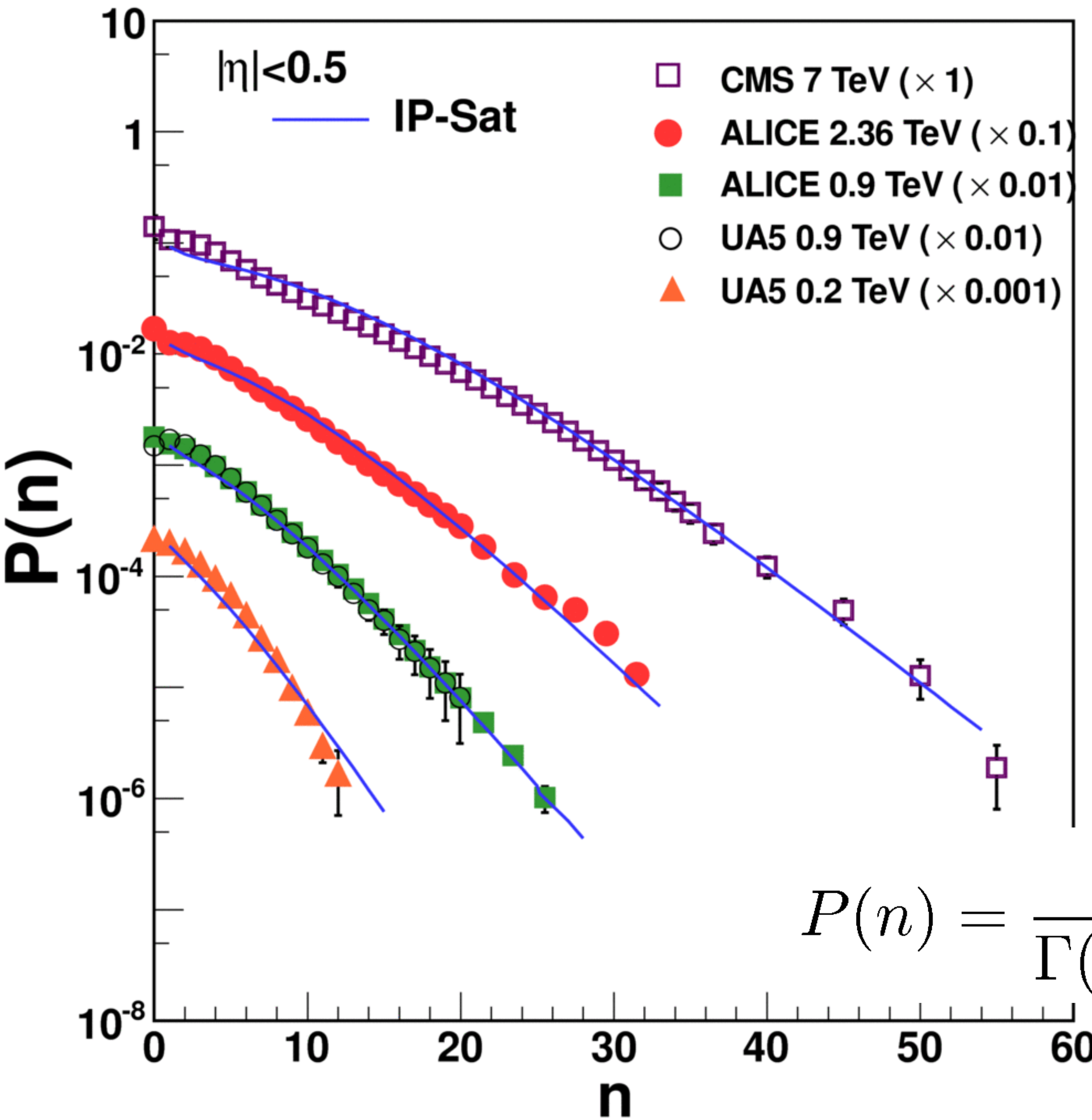
N_{part} fluctuations in p+Pb:



“intrinsic” particle production fluctuations

Multiplicity distributions in pp,
KNO scaling

Multiplicity distributions in *pp* collisions



P(n): negative binomial distribution

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\langle n \rangle^n k^k}{(\langle n \rangle + k)^{n+k}}$$

Tribedy & Venugopalan: arXiv:1112.2445

KNO scaling:

Koba, Nielsen, Olesen, NPB 40 (1972) 317

$\bar{n}\,P(n) \equiv \psi(z)$ is **universal** (independent of energy); $z \equiv n/\bar{n}$

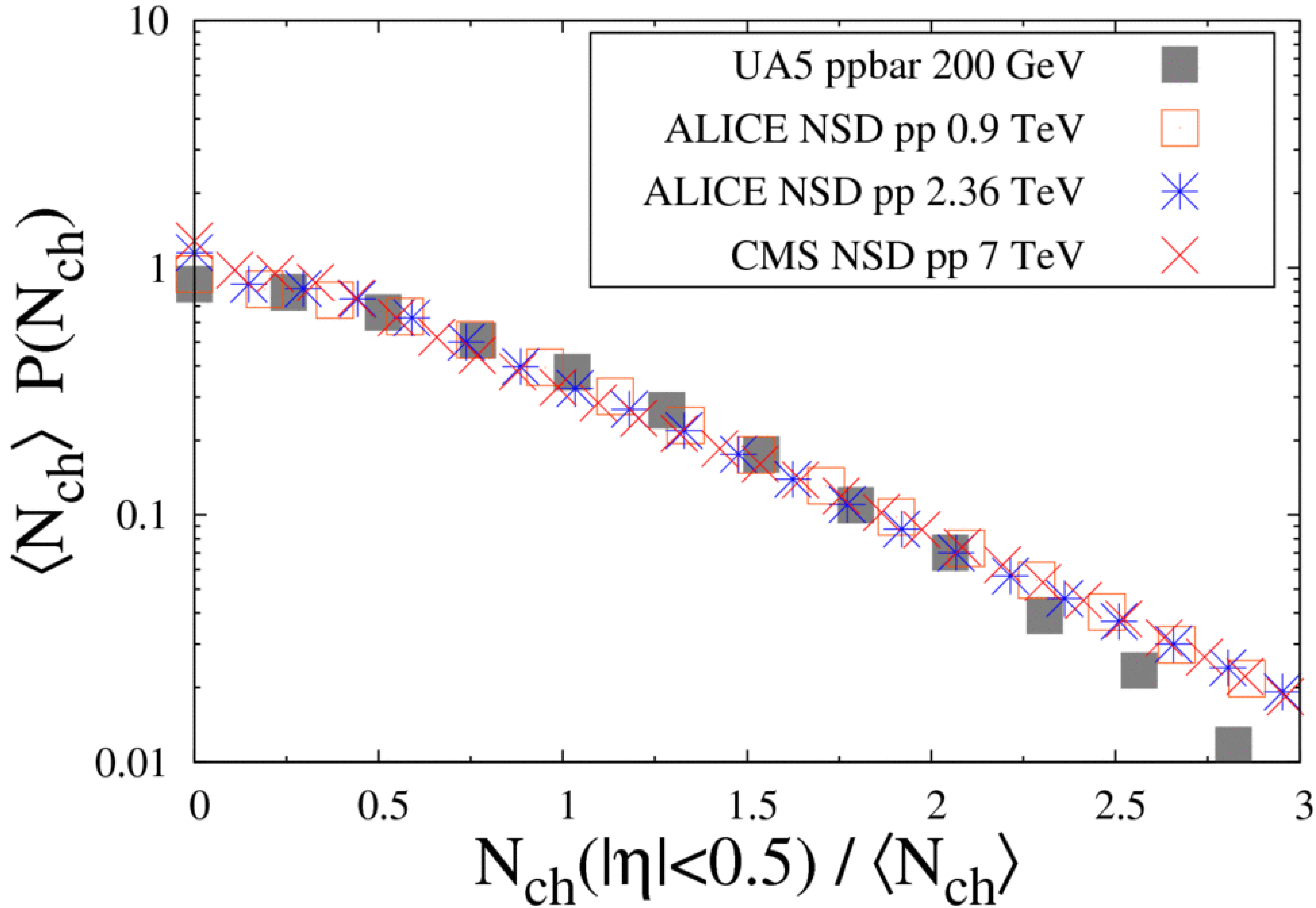
Note that if $k \ll \bar{n}$, NBD can be written as

$$\bar{n}\,P(n)\,dz \sim z^{k-1}\,e^{-kz}\,dz \qquad , \quad z \equiv n/\bar{n}$$

So, if k=const, this leads to KNO scaling !

for our fit to pp @ LHC: $k / \bar{n} \sim 0.16$ at 2360 GeV

KNO scaling in pp data



NBD from MV model; dominant contractions:

Gelis, Lappi, McLerran: arXiv:0905.3234

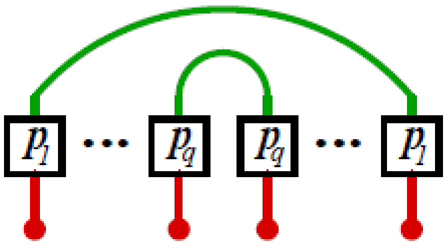
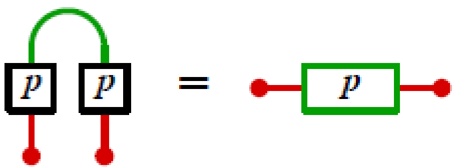
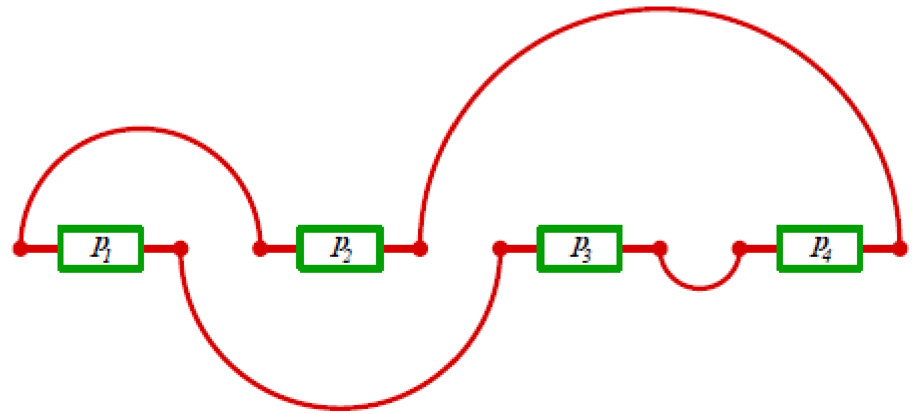


Figure 5: Rainbow diagram.



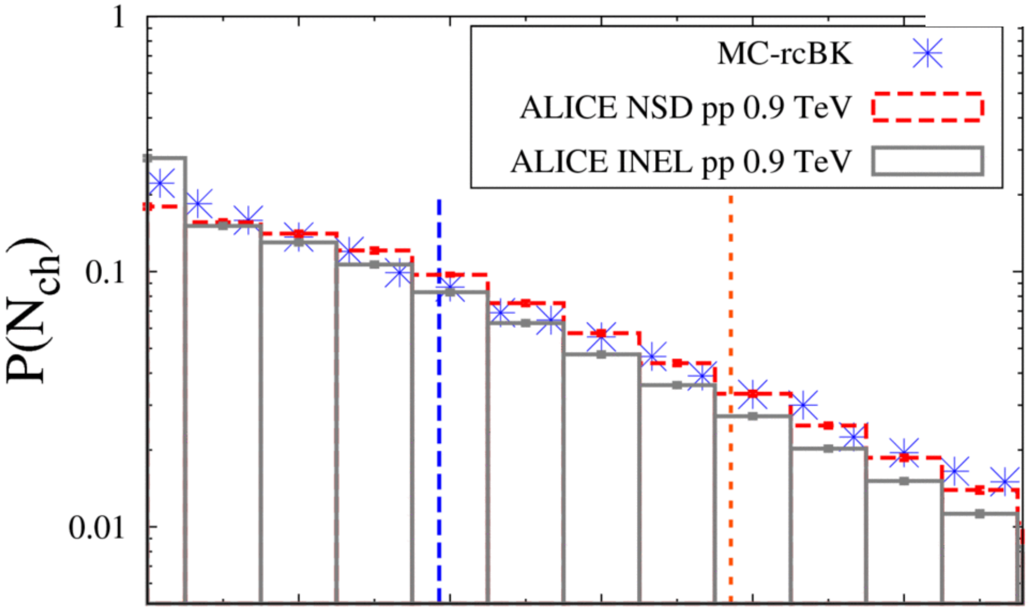


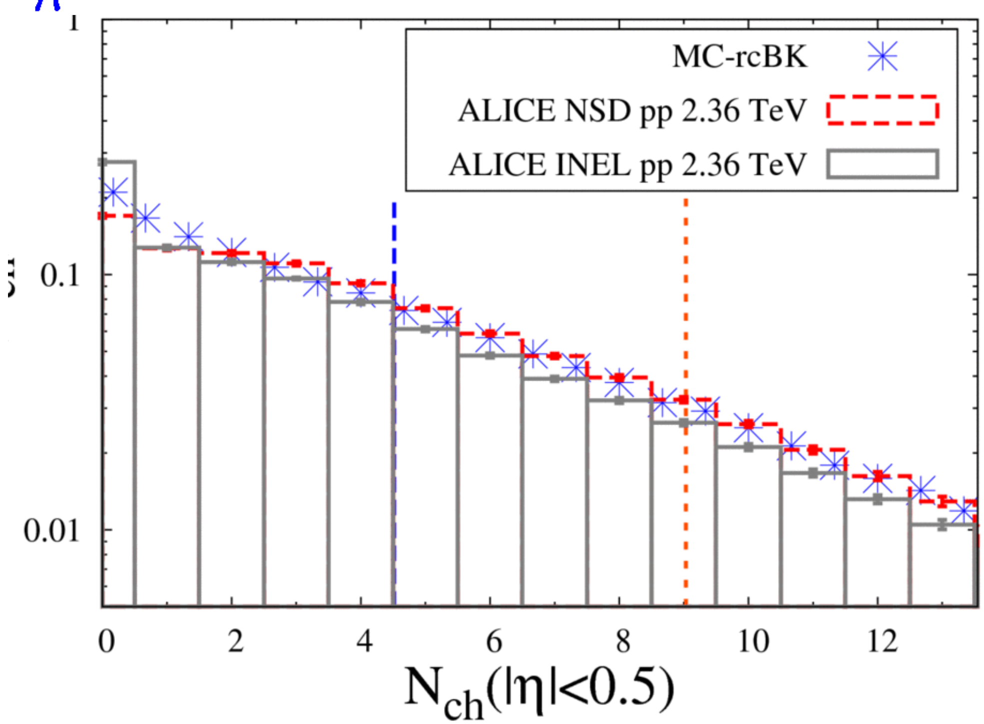
$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\langle n \rangle^n k^k}{(\langle n \rangle + k)^{n+k}}$$

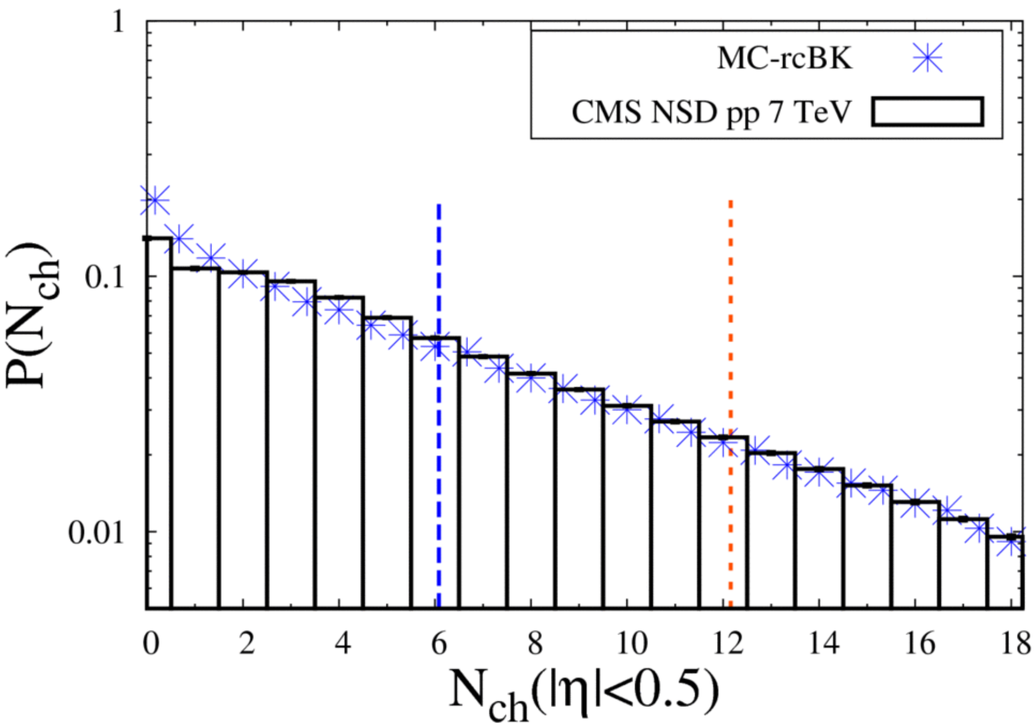
$$k \sim Q_{s0}^2 \sim T_A$$

Note: large k ↔ *little* fluctuations (~Poisson) !

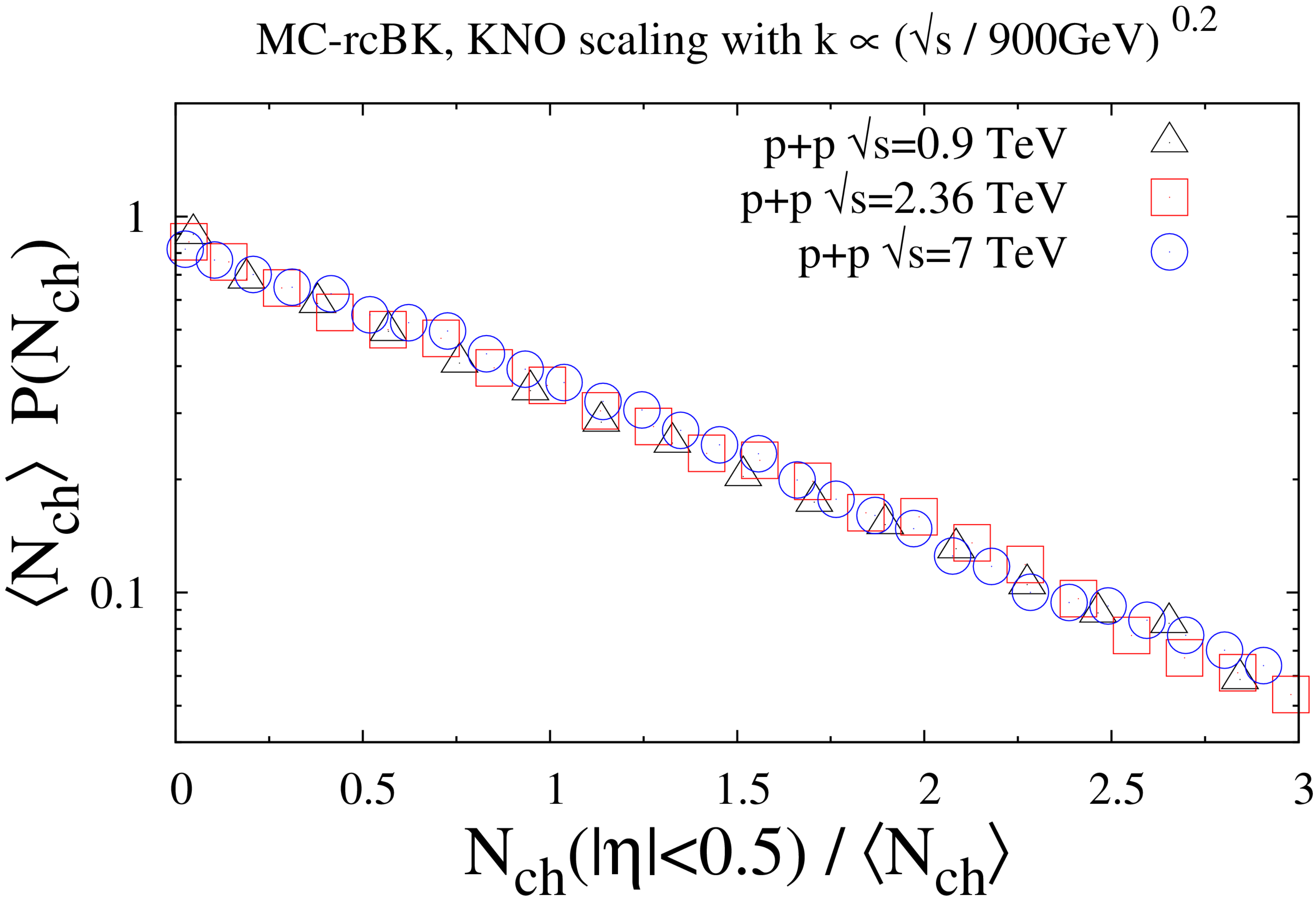
our result for constant $k = \frac{1}{\pi} \Delta x_\perp^2 \Delta\eta \, \Lambda_{\rm QCD}^2$



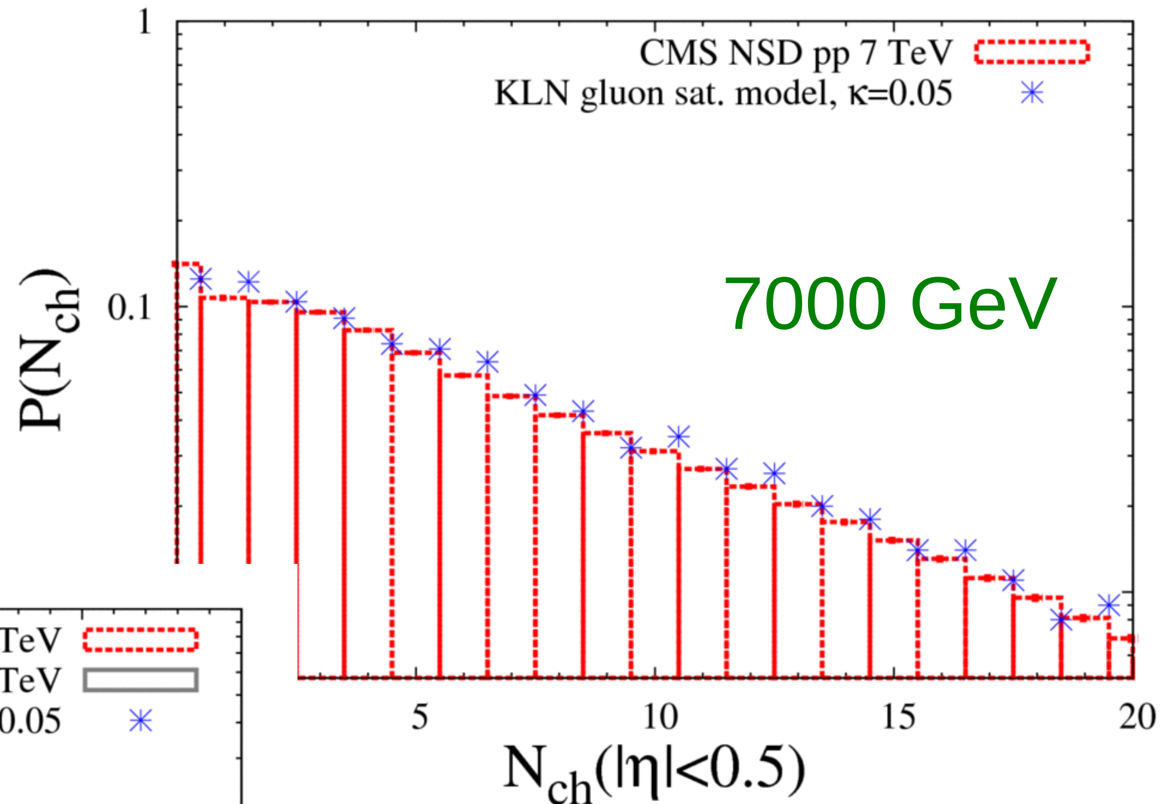


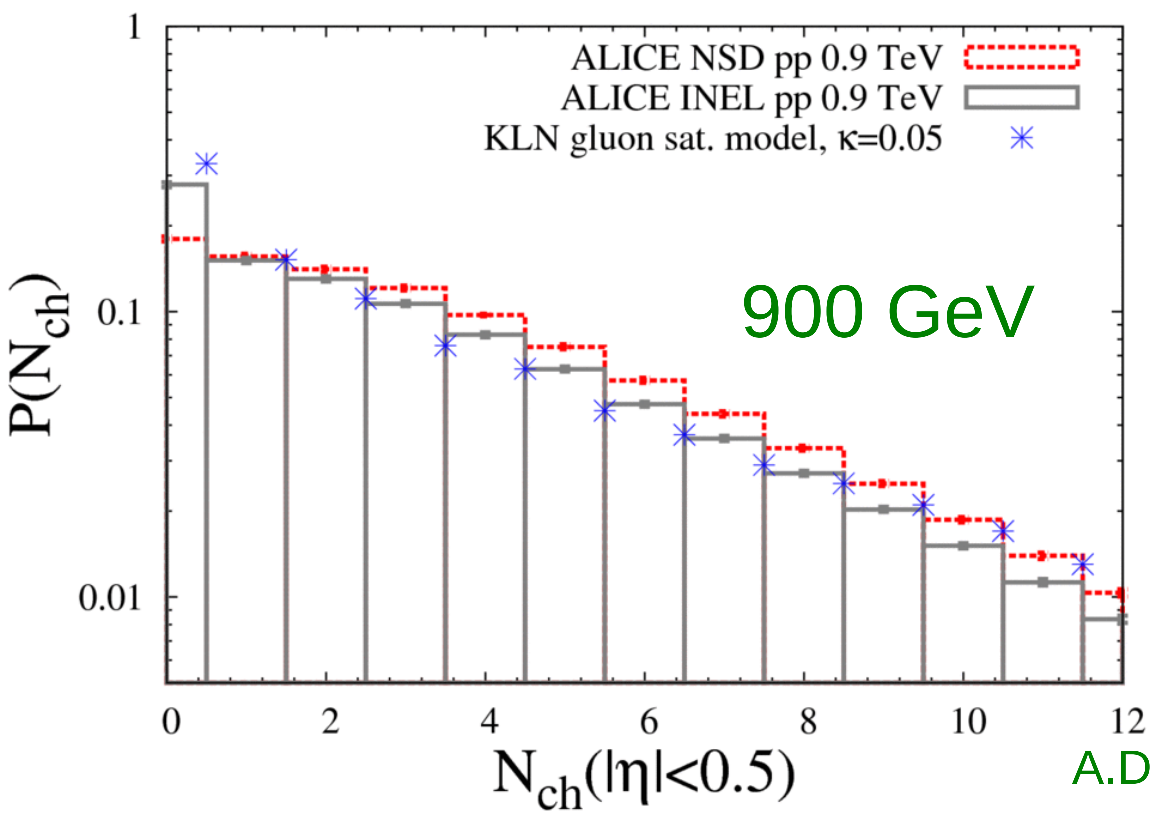


energy dependent k ~ $E_{CM}^{0.2}$



stronger energy dependence $k \sim Q_s^2(x)\, \sigma_{in}(s)$



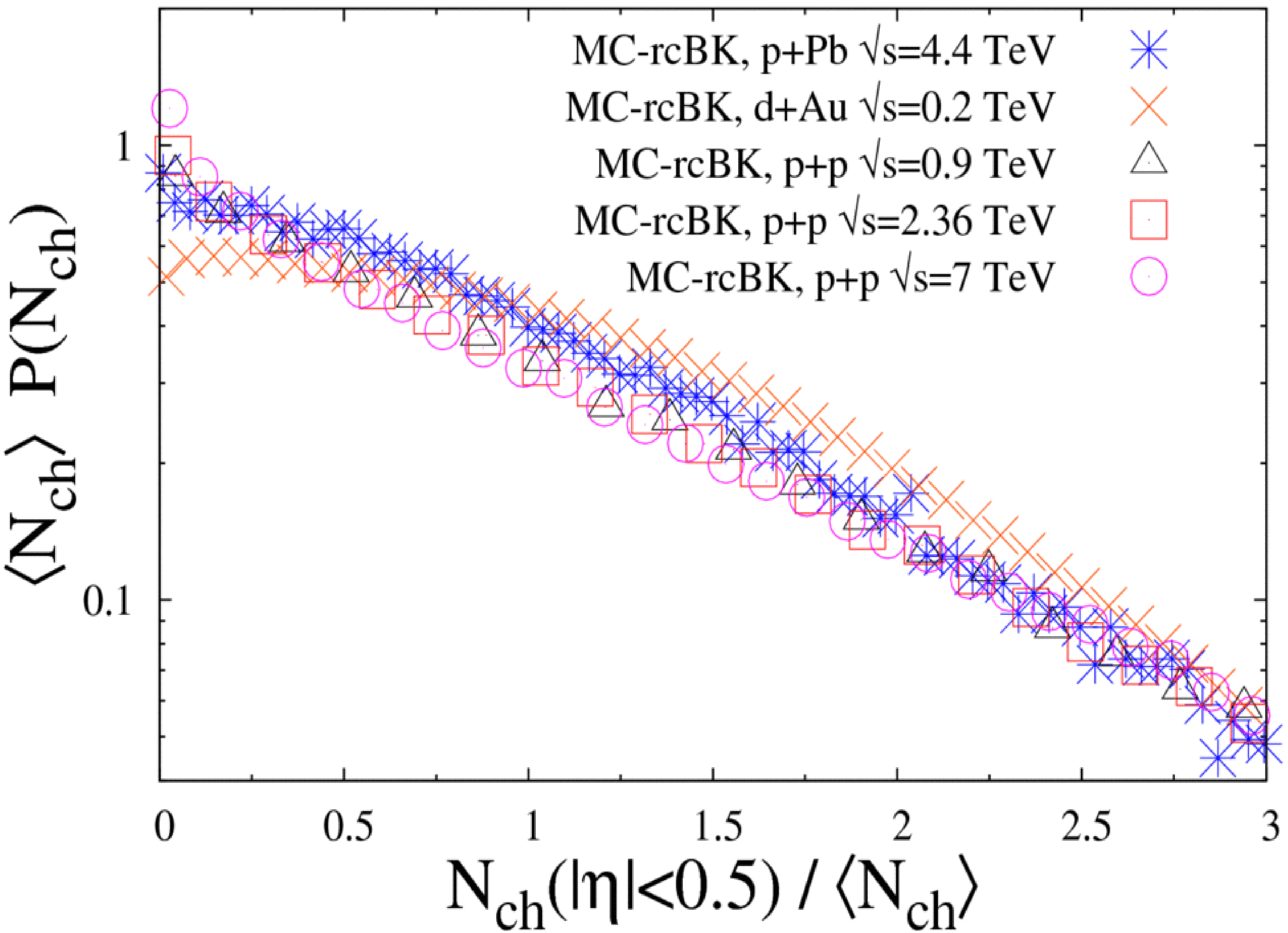


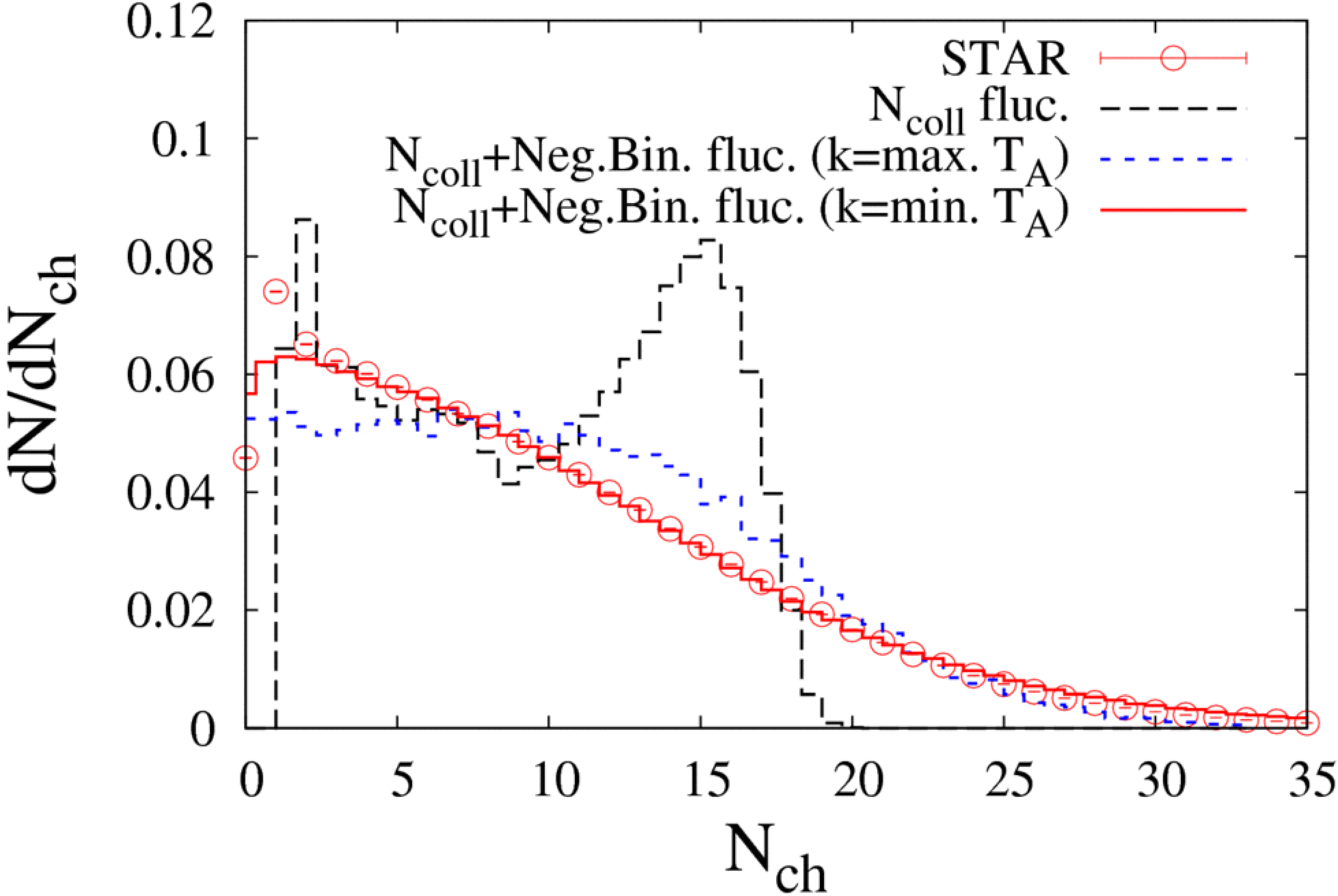
k_{900} a bit too small

A.D., Kharzeev, Levin, Nara arXiv:1111.3031

KNO scaling (even p+Pb approx.; prediction)

for A+B : $k_{AB} \sim k_{pp} \min(T_A, T_B)$





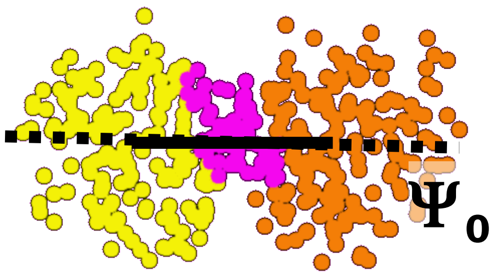
STAR
N$_{coll}$ fluc.
N$_{coll}$+Neg.Bin. fluc. (k=max. T$_A$)
N$_{coll}$+Neg.Bin. fluc. (k=min. T$_A$)
dN/dN$_{ch}$
N$_{ch}$
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
35

Eccentricity fluctuations

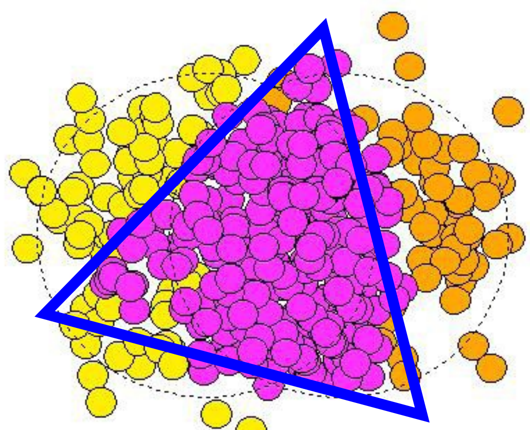
Event-by-event fluctuations in the shape of the initial collision zone may be important.

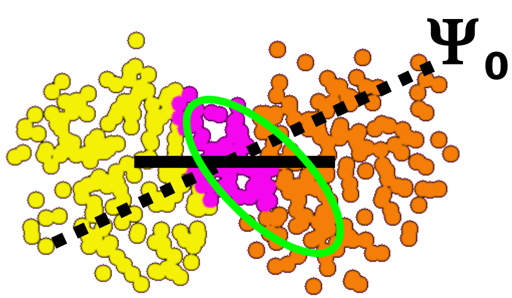
Specifically fluctuations in the nucleon positions.

B Alver and G Roland, Phys. Rev. C 81, 054905 (2010)



$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$





$$\langle \epsilon_{part} \rangle = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$

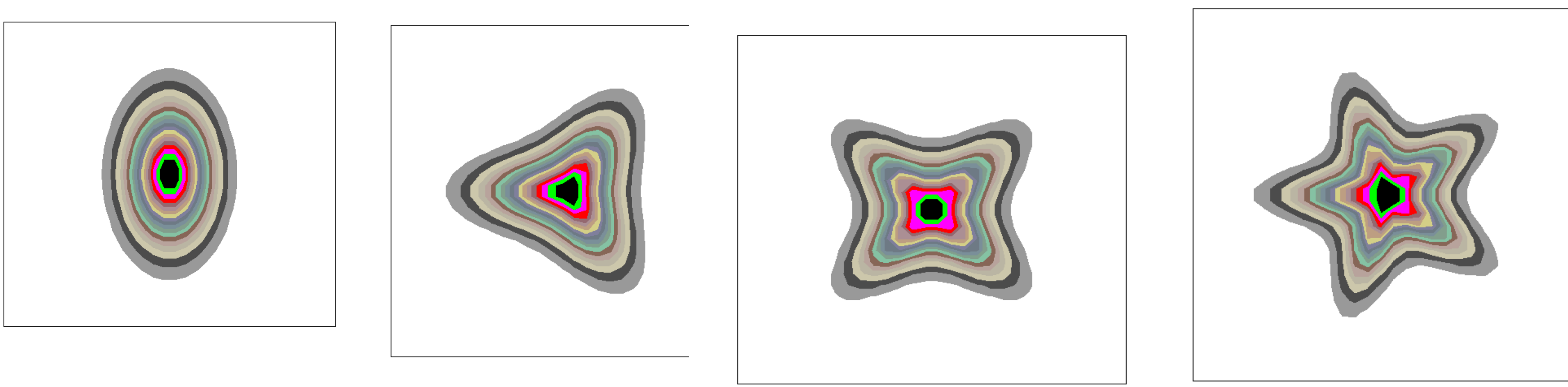
$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2, \quad \sigma_y^2 = \langle y^2 \rangle - \langle y \rangle^2, \quad \sigma_{xy}^2 = \langle xy \rangle - \langle y \rangle \langle x \rangle$$

R.S.Hollis (PHOBOS) nucl-ex0707.0125

Higher order Eccentricities

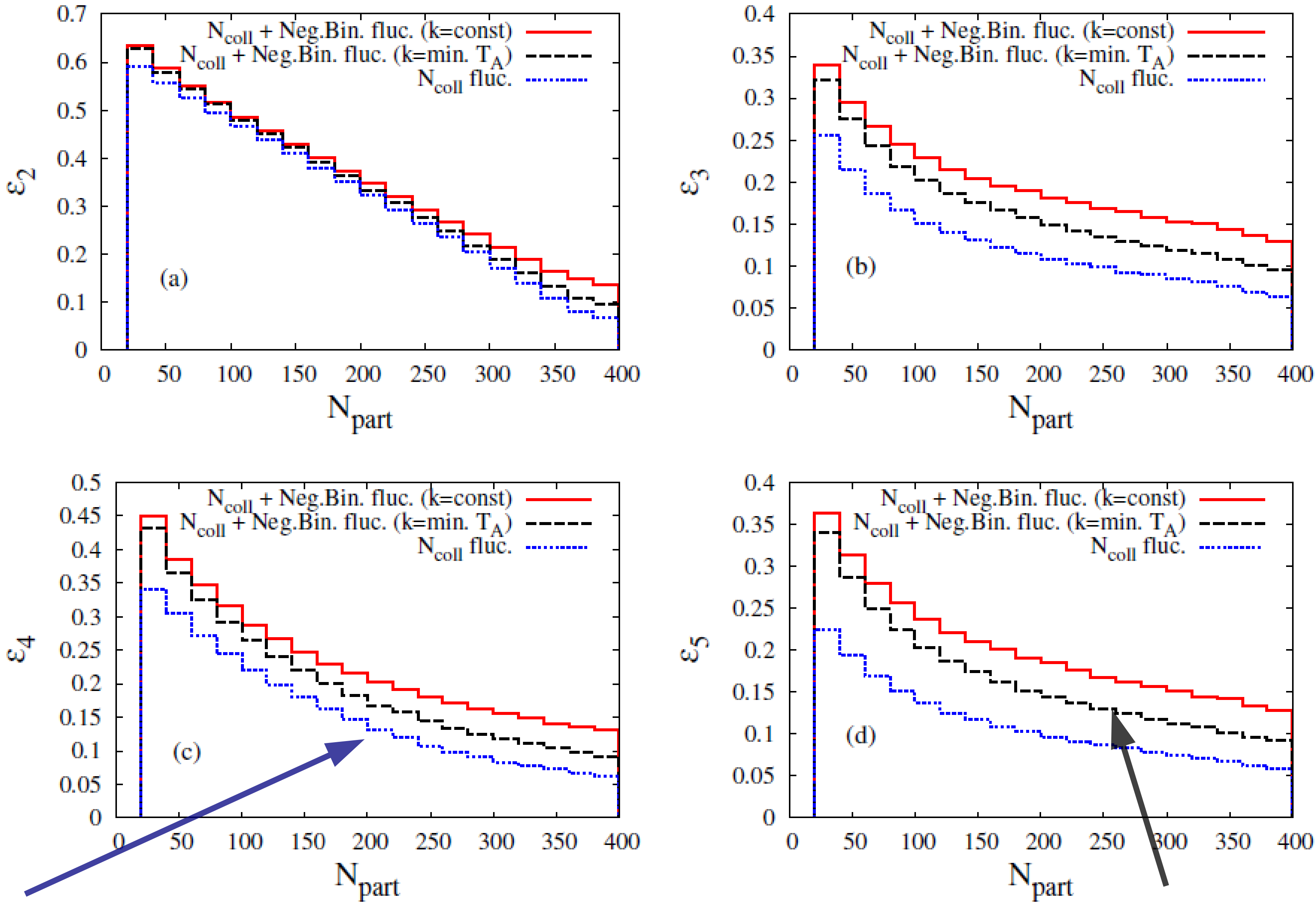
$$\epsilon_n = \frac{\sqrt{\langle r^2 \cos(n\phi) \rangle^2 + \langle r^2 \sin(n\phi) \rangle^2}}{\langle r^2 \rangle}$$

$$r^2 = x^2 + y^2,\ x = r\cos(\phi),\ y = r\sin(\phi)$$



$$e(r,\phi) = e_0 \exp\left[-\frac{r^2}{2\sigma^2}(1 + \epsilon_n \cos(n\phi))\right]$$

Eccentricities ε_n in Au+Au



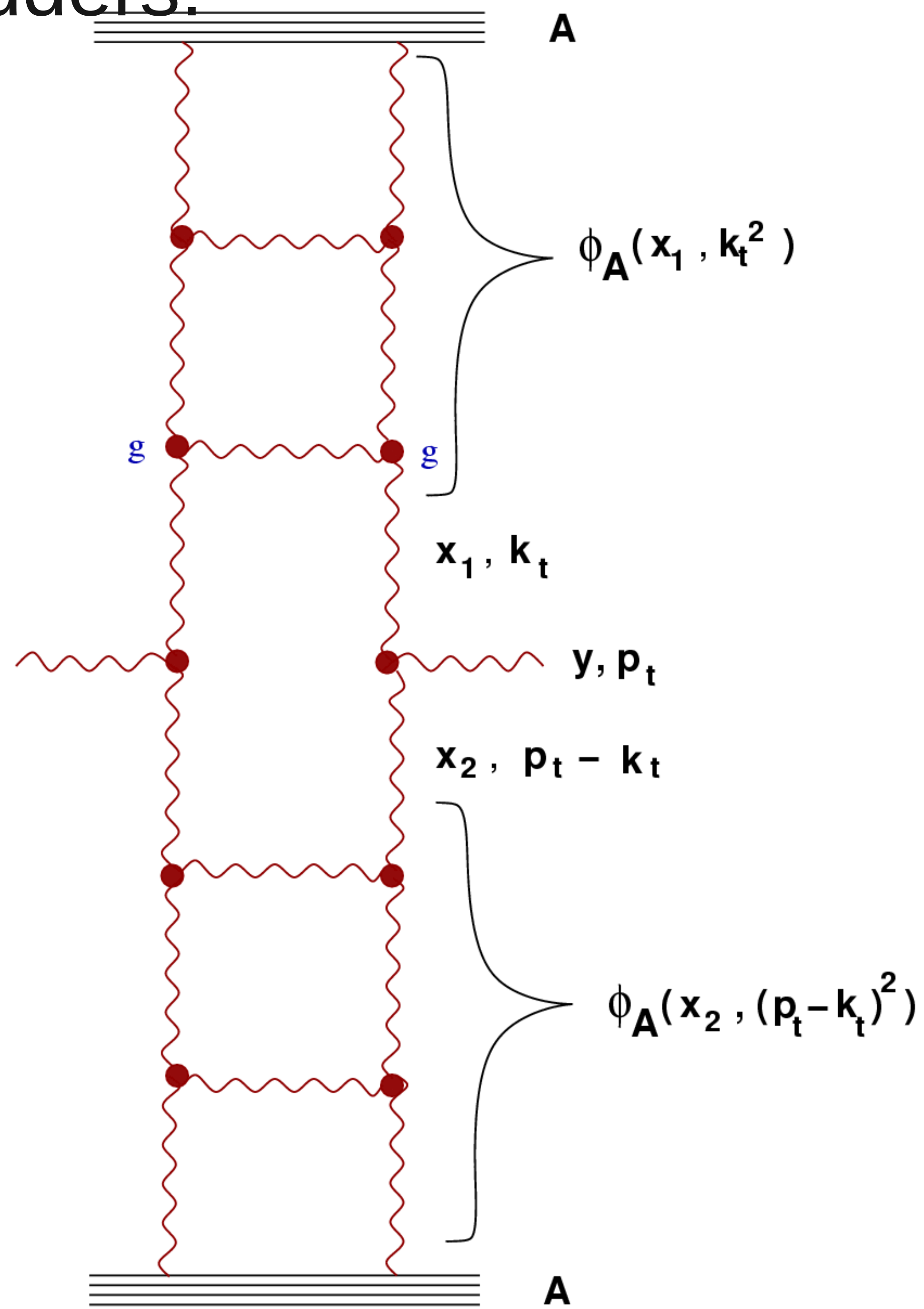
Glauber fluc only

+ neg. bin.
k ~ min(T_A,T_B)

Questions for theory...

Fluctuations in evolution ladders:

do (rapidity-enhanced)
quantum fluctuations
satisfy KNO scaling ?



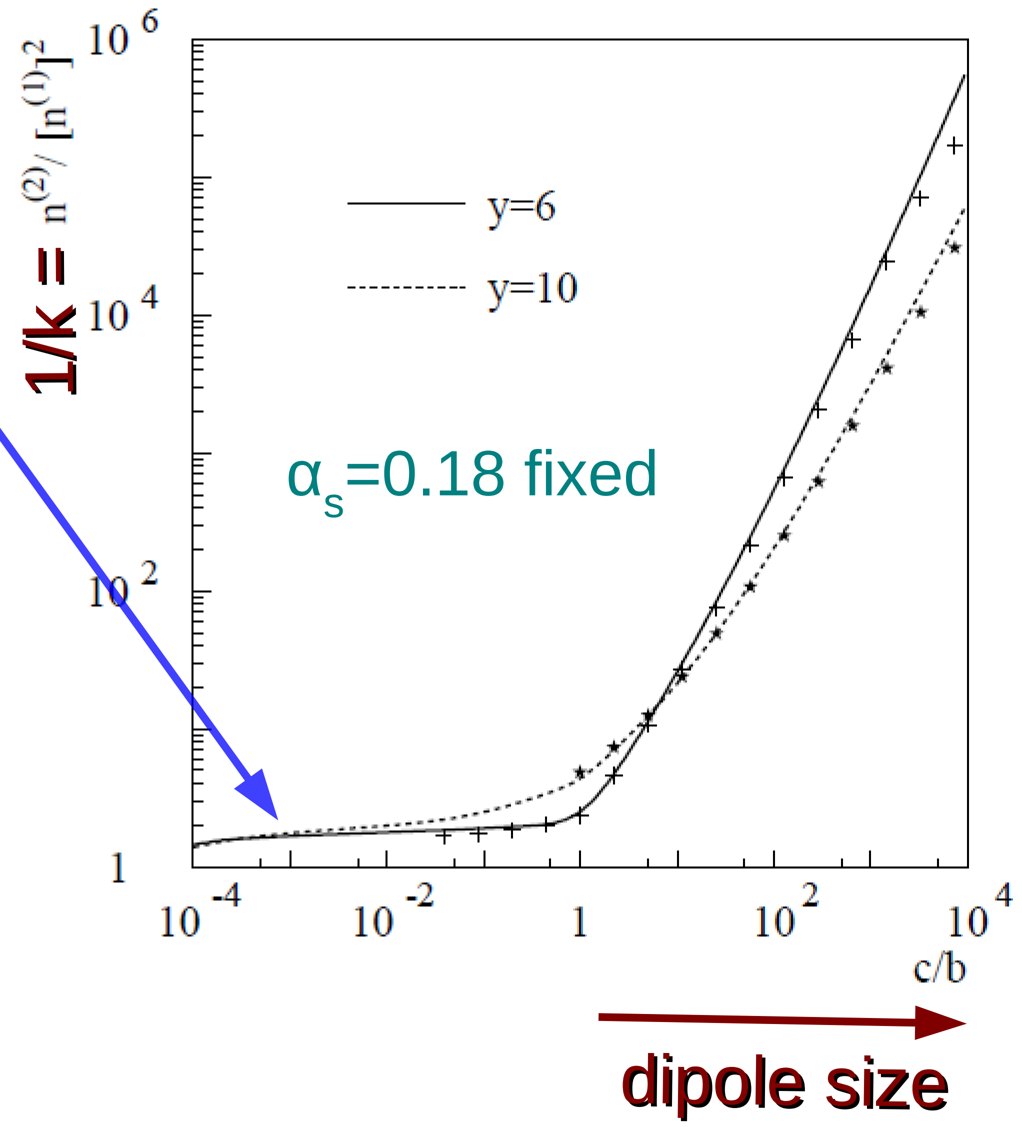
G.P. Salam, hep-ph/9504284

- KNO scaling for very small dipole sizes (DLA approx.)
- BFKL saddle point though gives

$$k^{-1} \equiv \beta_2 \sim e^{\# \bar{\alpha}_s y}$$

for small dipole sizes

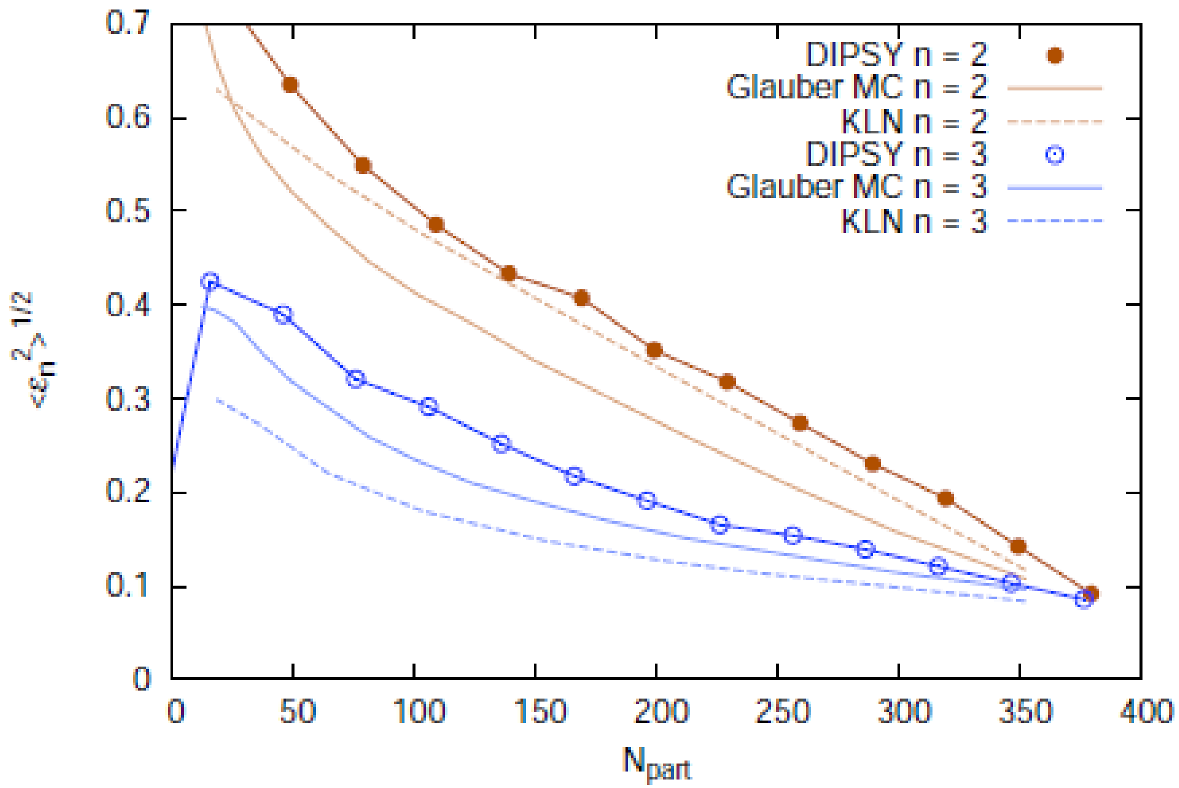
- "The main results are that for intermediate and large sizes, KNO scaling is not observed"



DIPSY MC w. fluctuations in BFKL ladders

Results: $\varepsilon_2, \varepsilon_3$

C. Flensburg: ISMD 2011, Hiroshima



compare DIPSY <-----> MC-KLN:
ε_2 similar, ε_3 much larger

- does DIPSY reproduce KNO in pp, pA ?

Non-Gaussian initial conditions for high-energy evolution

- Odderon operator $-d^{abc}\rho^a\rho^b\rho^c/\kappa_3$

S. Jeon and R. Venugopalan,
Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- Effective action for a system of $k \sim N_c A^{1/3} \gg 1$ valence quarks in SU(3);
- Random walk of SU(3) color charges in the space of representations (m,n);

- Probability $P(m,n) = e^{-S(m,n)}$

$$S(m,n;k) \simeq \frac{N_c}{k} C_2(m,n) - \frac{1}{3}\left(\frac{N_c}{k}\right)^2 C_3(m,n) + \frac{1}{6}\left(\frac{N_c}{k}\right)^3 C_4(m,n)$$

$C_2,\ C_3,\ C_4$ - Casimir operators for the representation (m,n)

- Define color charge per unit area $\rho^a \equiv g\,Q^a/\Delta^2 x$
  where $|Q| = \sqrt{Q^a Q^a} \equiv \sqrt{C_2}$

Beyond MV action...

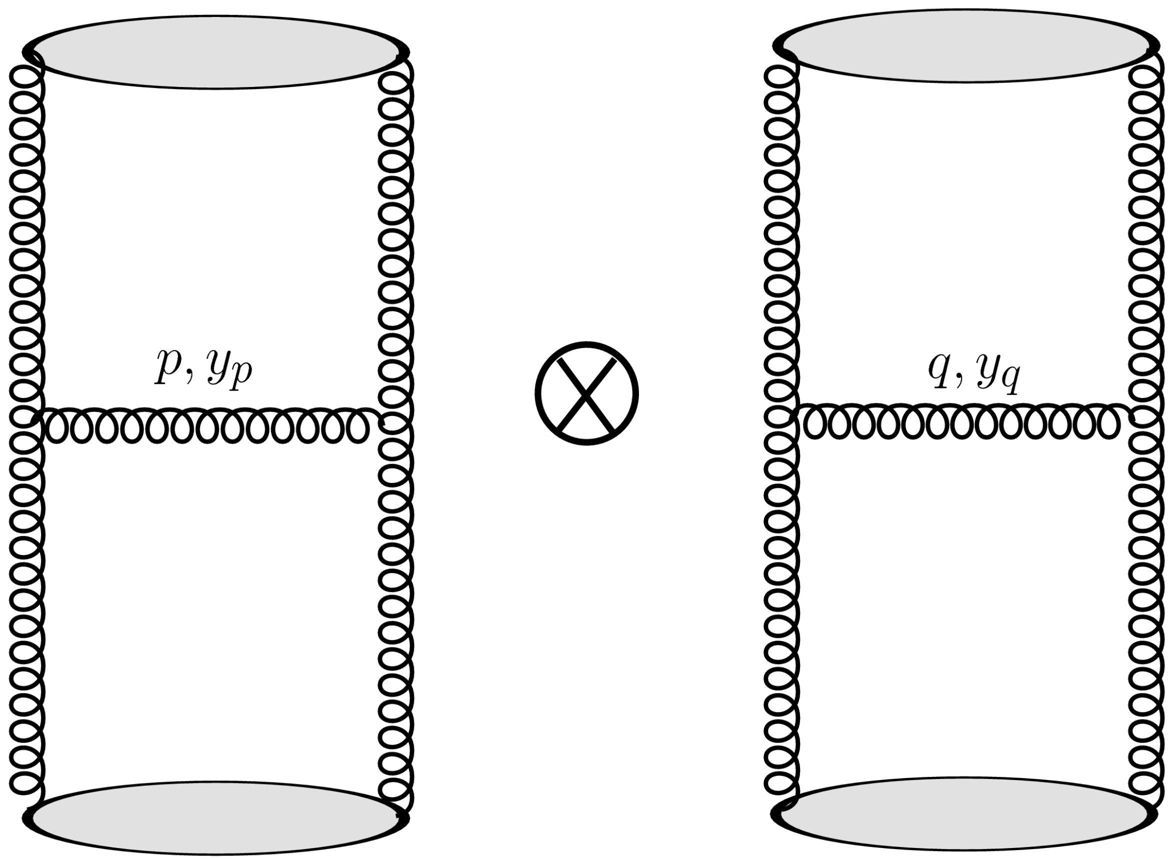
Elena Petreska et al: PRD 2011

$$S = \int d^2x_\perp \left\{ \frac{1}{2\mu^2}\rho^a\rho^a - \frac{1}{\kappa_3} d^{abc}\rho^a\rho^b\rho^c + \right.$$
$$\left[\frac{1}{\kappa_4}\left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right) + \right.$$
$$\left. \left. \frac{1}{\kappa_4'}\left(d^{abe}d^{cde} + d^{ace}d^{bde} + d^{ade}d^{bce}\right)\right] \rho^a\rho^b\rho^c\rho^d \right\}$$

+ soft YM fields + coupling of soft ↔ hard

- last term absent for SU(2) and SU(3)
- $\mu^2 \sim g^2 A^{1/3}$; $\kappa_3 \sim g^3 A^{2/3}$; $\kappa_4 \sim g^4 A$

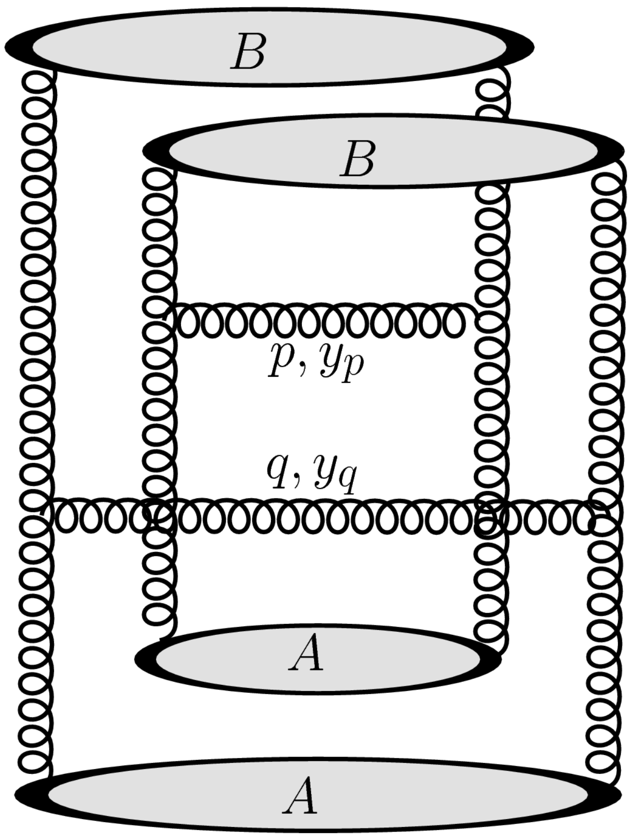
independent 2-gluon production with MV action



$$
\sim \quad -N_c^2(N_c^2-1)^2\,(\pi R^2)^2\,\frac{\mu^8}{p^2\,q^2}\int\frac{dk^2}{k^2}\,\frac{1}{(p-k)^2}\int\frac{dk'^{\,2}}{k'^{\,2}}\,\frac{1}{(q-k')^2}
$$

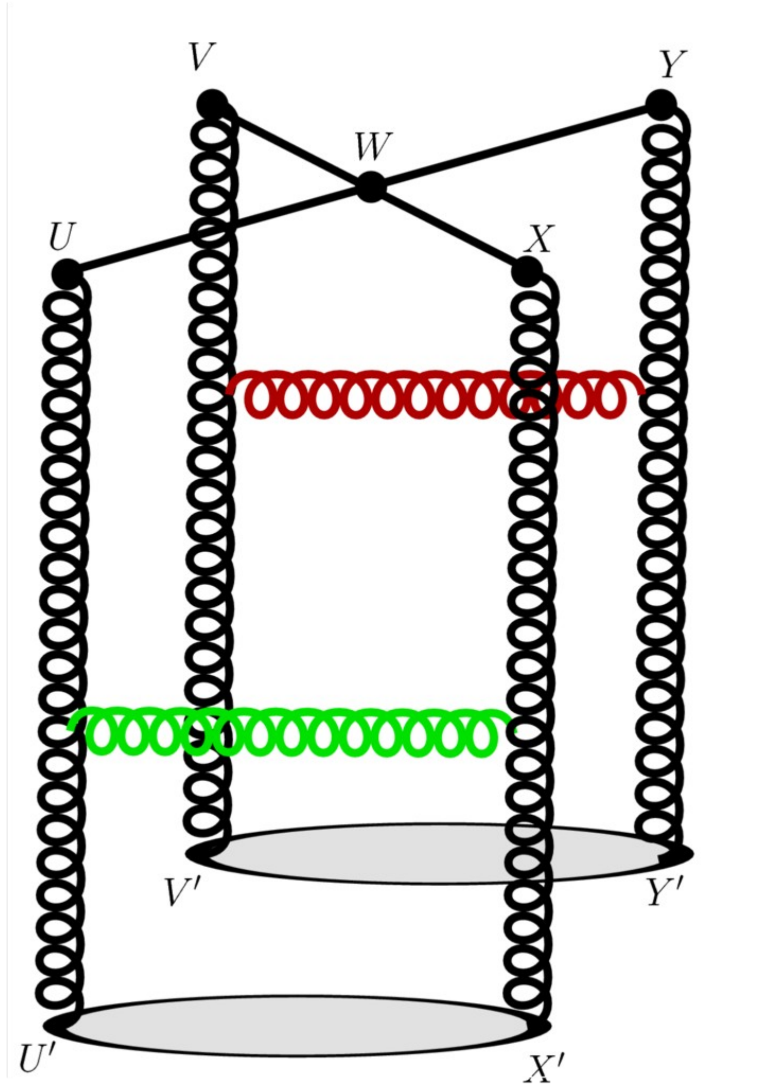
$$
\sim \quad -\frac{N_c^2(N_c^2-1)^2}{p^4q^4}\,A^{8/3}\log\frac{p^2}{Q_s^2}\,\log\frac{q^2}{Q_s^2}
$$

double gluon production with MV action



$$\sim \; N_c^2(N_c^2-1)\,\pi R^2\,\frac{(\mu^2)^4}{p^2q^2}\int\frac{dk^2}{k^4}\frac{1}{(p-k)^2}\frac{1}{(q-k)^2}$$

$$\sim \; \frac{N_c^2(N_c^2-1)}{p^4q^4}\,A^{5/3}$$



$$
\begin{aligned}
\sim \quad & -N_c^2 (N_c^2 - 1)^2 \, \frac{(\mu^2)^2 \, (\mu^4)^2}{\kappa_4} \, \frac{\pi R^2}{p^2 \, q^2} \int \frac{dk^2}{k^2} \frac{1}{(p-k)^2} \int \frac{dk'^{\,2}}{k'^{\,2}} \frac{1}{(q-k')^2} \\
\sim \quad & - \frac{N_c^2 (N_c^2 - 1)^2}{p^4 q^4} \, A \, \log \frac{p^2}{Q_s^2} \, \log \frac{q^2}{Q_s^2}
\end{aligned}
$$

compute k^{-1} from 2-particle connected diagrams:

E. Petreska + A.D. 2012

$$Q_{s0}^2 \, S_\perp \, \frac{1}{k} \simeq \frac{2\pi}{N_c^2 - 1} - \frac{\#}{A^{2/3}}$$

MV ρ^4

“#” should be small so as not to ruin KNO scaling in pp or pA !

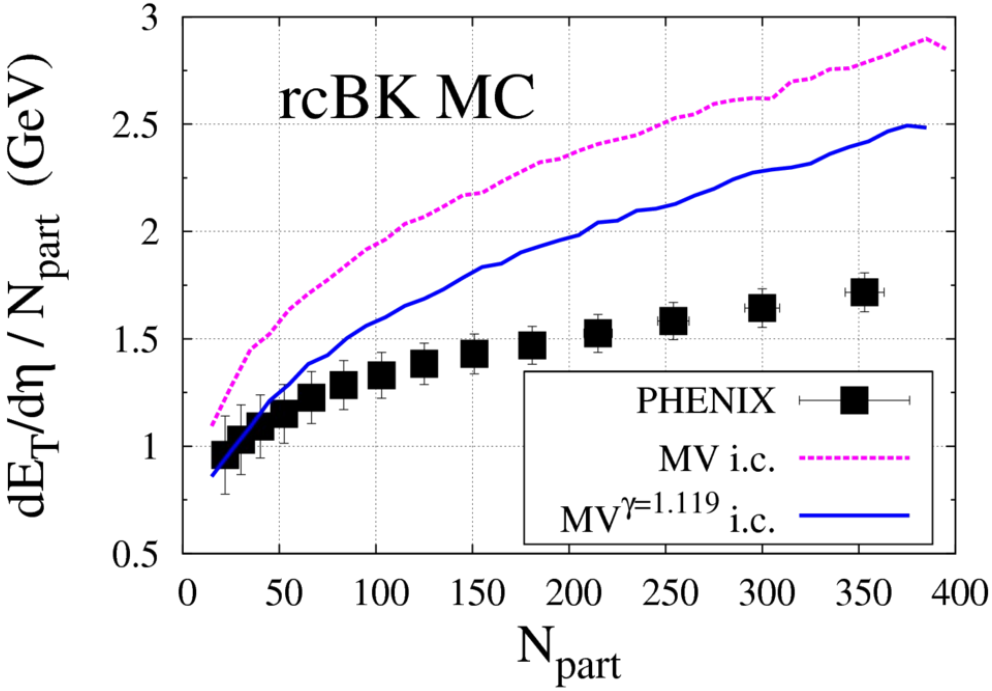
We'll know soon.

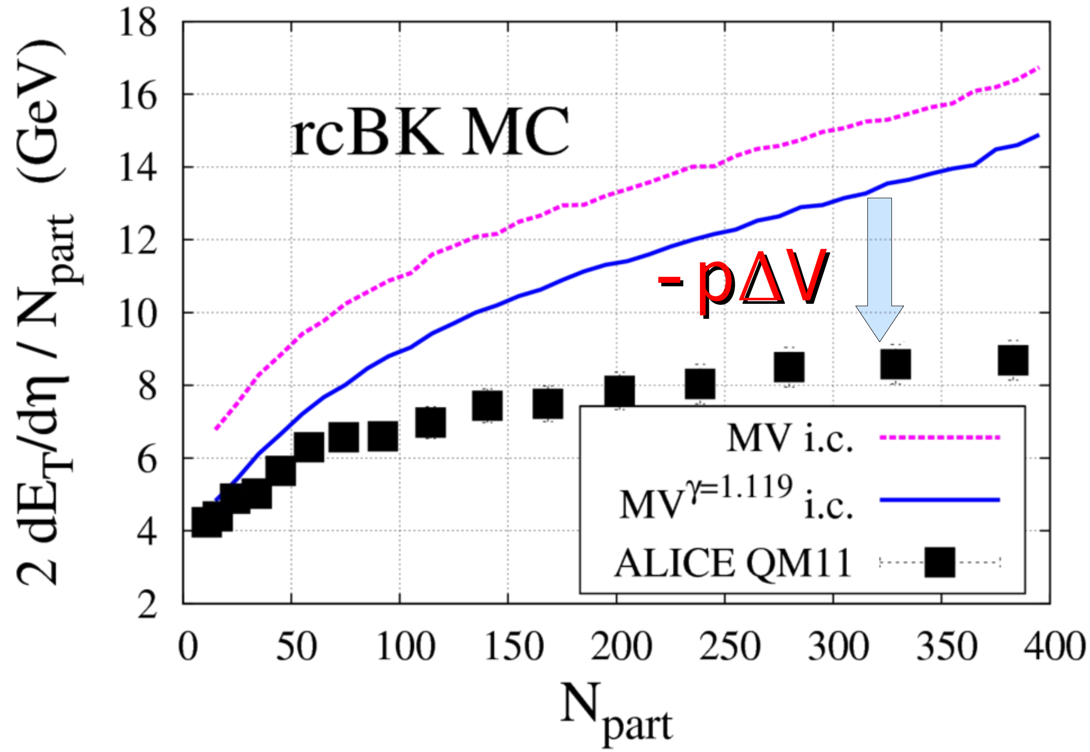
Summary

- multiplicity distributions in pp @ LHC exhibit KNO scaling (η=0, $n/\bar{n}$ <~ 3)
- can be described by NBD with k « $\bar{n}$
- approx. KNO scaling predicted for p+Pb @ LHC (KNO flucs dominate over Glauber flucs)
- higher-order eccentricities ε_3 etc. in HIC dominated by Glauber geometry flucs but increase substantially
-
- more theoretical studies of fluctuations definitely needed
  - constrain couplings of higher ρ^n operators
  - evolution with energy

Backup Slides

Back to AA : centrality and energy dependence of $E_\perp$

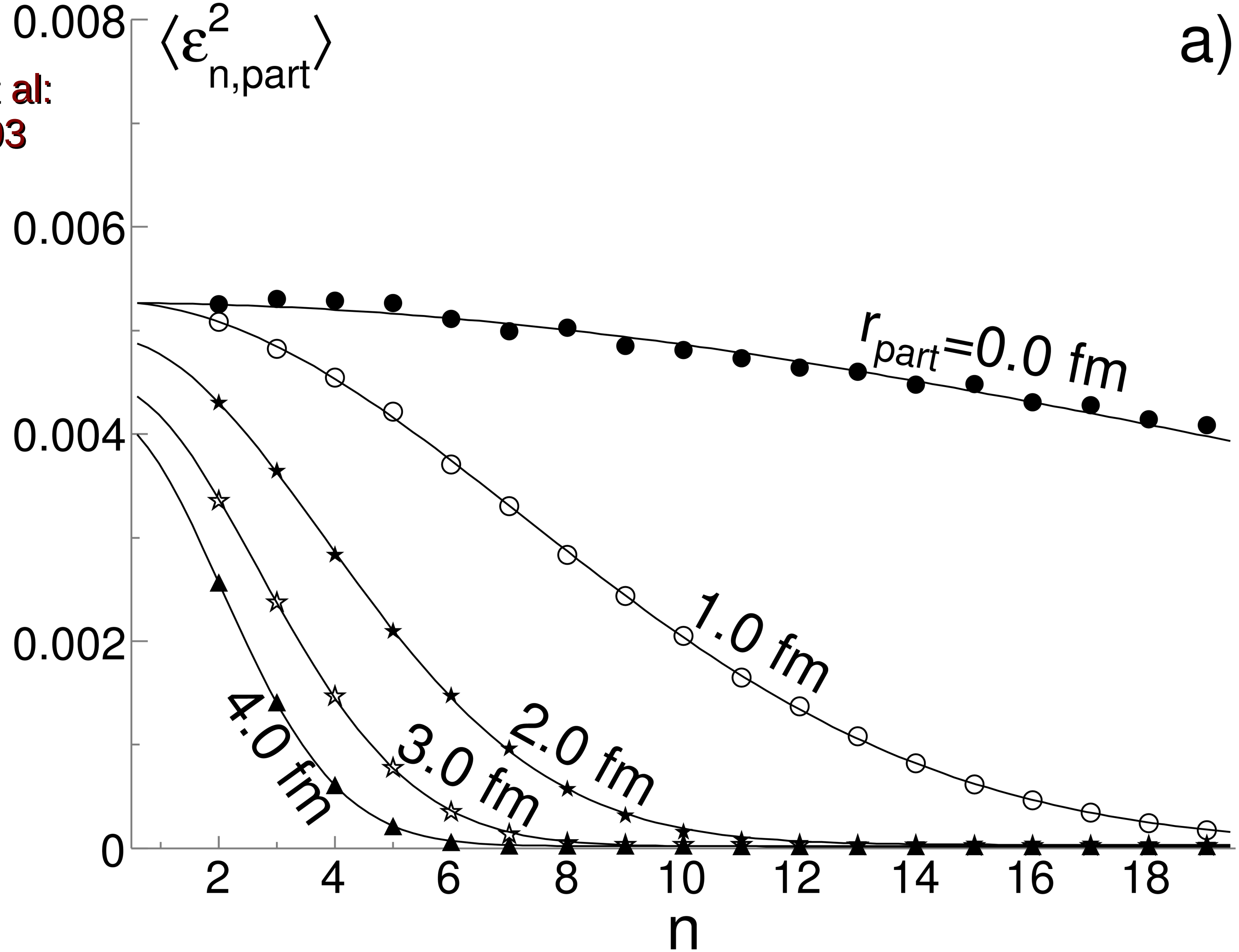




- (again, no g ⟶ h multiplication factor κ here)
- 1d ideal hydro: $E_\perp^f / E_\perp^i \approx T_f / T_i \approx 1/2$
- interesting: $(dE_\perp/d\eta) / (A \sqrt{s}) \approx 0.5\%$ at LHC_{2760}, centrl Pb+Pb !

Higher-order moments and the length scale of fluctuations

P. Sorensen et al: arXiv:1102.1403



Y. Nara: ISMD 2011